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$e^{\sin x}$ may be similarly developed.

$$e^{\sin x} = a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_8x^8 + \cdots. \qquad (2)$$

Differentiating,

$$e^{\sin x}\cos x = a_1 + 2a_2x + 3a_3x^2 + 4a_4x^3 + \cdots + 8a_8x^7 + \cdots$$
$$= (a_0 + a_1x + a_2x^2 + a_3x^3 + \cdots + a_8x^8 + \cdots)\left(1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \cdots\right).$$

Equating coefficients,

$$a_1 = a_0, \quad 2a_2 = a_1, \quad 3a_3 = a_2 - \frac{a_0}{2!}, \quad 4a_4 = a_3 - \frac{a_1}{2!}, \quad 5a_5 = a_4 - \frac{a_2}{2!} + \frac{a_0}{4!},$$

$$6a_6 = a_5 - \frac{a_3}{2!} + \frac{a_1}{4!}, \quad 7a_7 = a_6 - \frac{a_4}{2!} + \frac{a_2}{4!} - \frac{a_0}{6!}, \quad 8a_8 = a_7 - \frac{a_5}{2!} + \frac{a_3}{4!} - \frac{a_1}{6!}.$$

Making $x = 0$ in equation (2) gives $a_0 = 1$ and the preceding equations give

$$a_1 = 1, \quad a_2 = \tfrac{1}{2}, \quad a_3 = 0, \quad a_4 = -\tfrac{1}{8}, \quad a_5 = -\tfrac{1}{15}, \quad a_6 = -\tfrac{1}{240}, \quad a_7 = \tfrac{1}{90}, \quad a_8 = \tfrac{31}{5760}.$$

Hence,

$$e^{\sin x} = 1 + x + \frac{x^2}{2} - \frac{x^4}{8} - \frac{x^5}{15} - \frac{x^6}{240} + \frac{x^7}{90} + \frac{31x^8}{5760} + \cdots.$$

To develop $\tan x$, we notice that $\tan(-x) = -\tan x$. Hence, the expansion will contain only odd powers of x.

Assume

$$\tan x = a_1x + a_3x^3 + a_5x^5 + a_7x^7 + a_9x^9 + \cdots.$$

Differentiating,

$$\sec^2 x = a_1 + 3a_3x^2 + 5a_5x^4 + 7a_7x^6 + 9a_9x^8 + \cdots = 1 + \tan^2 x$$
$$= 1 + (a_1x + a_3x^3 + a_5x^5 + a_7x^7 + a_9x^9 + \cdots)^2.$$

Equating coefficients of like powers of x,

$$a_1 = 1, \quad 3a_3 = a_1^2, \quad 5a_5 = 2a_1a_3, \quad 7a_7 = 2a_1a_5 + a_3^2, \quad 9a_9 = 2a_1a_7 + 2a_3a_5.$$

From these, we obtain

$$a_1 = 1, \quad a_3 = \tfrac{1}{3}, \quad a_5 = \tfrac{2}{15}, \quad a_7 = \tfrac{17}{315}, \quad a_9 = \tfrac{62}{2835}.$$

Hence,

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \frac{62x^9}{2835} + \cdots.$$

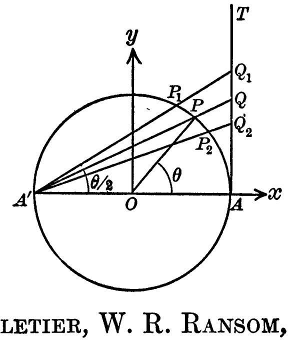
Also solved by E. D. Grant, H. L. Olson, J. L. Riley, and the Proposer.

2738 [January, 1919]. Proposed by W. D. CAIRNS, Oberlin College.

Prove that between any two points on a unit circle with its center at the origin there is a point whose coördinates are rational.

Solution by P. J. Daniell, Rice Institute.

Let $A'OA$ be the diameter on the x-axis, and let P_1, P_2 be the two given points. Through A draw AT perpendicular to $A'OA$. Let $A'P_1$, $A'P_2$ intersect AT in Q_1, Q_2. By the theory of irrational numbers between the points Q_1, Q_2 on AT there is a point Q such that AQ is rational and indeed equal to $2m/n$, where m, n are integers. Let $A'Q$ intersect the circle in P. Then P is the required point. It is assumed, and this involves no loss in generality, that P_1, P_2 lie on the same side of the x-axis. Then P lies between P_1 and P_2.



$$x = \cos\theta = \frac{1 - \tan^2\frac{\theta}{2}}{1 + \tan^2\frac{\theta}{2}} = \frac{1 - \frac{AQ^2}{A'A^2}}{1 + \frac{AQ^2}{A'A^2}} = \frac{1 - \frac{m^2}{n^2}}{1 + \frac{m^2}{n^2}} = \frac{n^2 - m^2}{n^2 + m^2}$$

is rational. Similarly $y = \dfrac{2mn}{n^2 + m^2}$ is rational.

Also solved by R. A. Johnson, H. L. Olson, A. Pelletier, W. R. Ransom, J. Rosenbaum, E. Swift, and the Proposer.